OCR
RECOGNISING ACHIEVEMENT

Monday 28 January 2013 – Morning

AS GCE MATHEMATICS

4732/01 Probability and Statistics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4732/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 When a four-sided spinner is spun, the number on which it lands is denoted by X, where X is a random variable taking values 2, 4, 6 and 8. The spinner is biased so that $P(X = x) = kx$, where k is a constant.

(i) Show that $P(X = 6) = \frac{3}{10}$. **[2]**

(ii) Find $E(X)$ and $Var(X)$. **[5]**

2 **(i)** Kathryn is allowed three attempts at a high jump. If she succeeds on any attempt, she does not jump again. The probability that she succeeds on her first attempt is $\frac{3}{4}$. If she fails on her first attempt, the probability that she succeeds on her second attempt is $\frac{3}{8}$. If she fails on her first two attempts, the probability that she succeeds on her third attempt is $\frac{3}{16}$. Find the probability that she succeeds. **[3]**

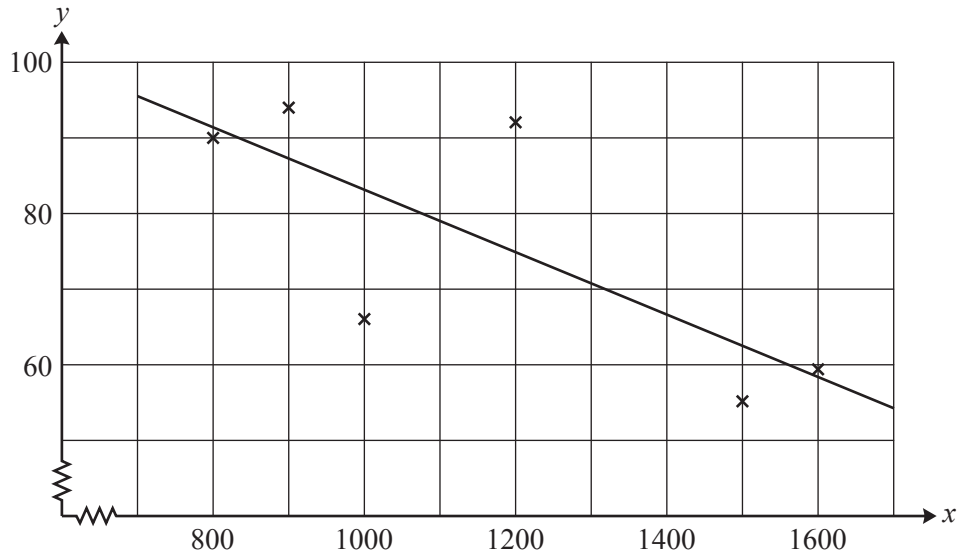
(ii) Khaled is allowed two attempts to pass an examination. If he succeeds on his first attempt, he does not make a second attempt. The probability that he passes at the first attempt is 0.4 and the probability that he passes on either the first or second attempt is 0.58. Find the probability that he passes on the second attempt, given that he failed on the first attempt. **[3]**

3 The Gross Domestic Product per Capita (GDP), x dollars, and the Infant Mortality Rate per thousand (IMR), y, of 6 African countries were recorded and summarised as follows.

$$n = 6 \qquad \Sigma x = 7000 \qquad \Sigma x^2 = 8\,700\,000 \qquad \Sigma y = 456 \qquad \Sigma y^2 = 36\,262 \qquad \Sigma xy = 509\,900$$

(i) Calculate the equation of the regression line of y on x for these 6 countries. **[4]**

The original data were plotted on a scatter diagram and the regression line of y on x was drawn, as shown below.



(ii) The GDP for another country, Tanzania, is 1300 dollars. Use the regression line in the diagram to estimate the IMR of Tanzania. **[1]**

(iii) The GDP for Nigeria is 2400 dollars. Give two reasons why the regression line is unlikely to give a reliable estimate for the IMR for Nigeria. **[2]**

(iv) The actual value of the IMR for Tanzania is 96. The data for Tanzania ($x = 1300$, $y = 96$) is now included with the original 6 countries. Calculate the value of the product moment correlation coefficient, r, for all 7 countries. **[4]**

(v) The IMR is now redefined as the infant mortality rate per hundred instead of per thousand, and the value of r is recalculated for all 7 countries. Without calculation state what effect, if any, this would have on the value of r found in part **(iv)**. **[1]**

4 **(i)** How many different 3-digit numbers can be formed using the digits 1, 2 and 3 when

(a) no repetitions are allowed, **[1]**

(b) any repetitions are allowed, **[2]**

(c) each digit may be included at most twice? **[2]**

(ii) How many different **4-digit** numbers can be formed using the digits 1, 2 and 3 when each digit may be included at most twice? **[5]**

5 A random variable X has the distribution $\mathrm{B}(5, \frac{1}{4})$.

(i) Find

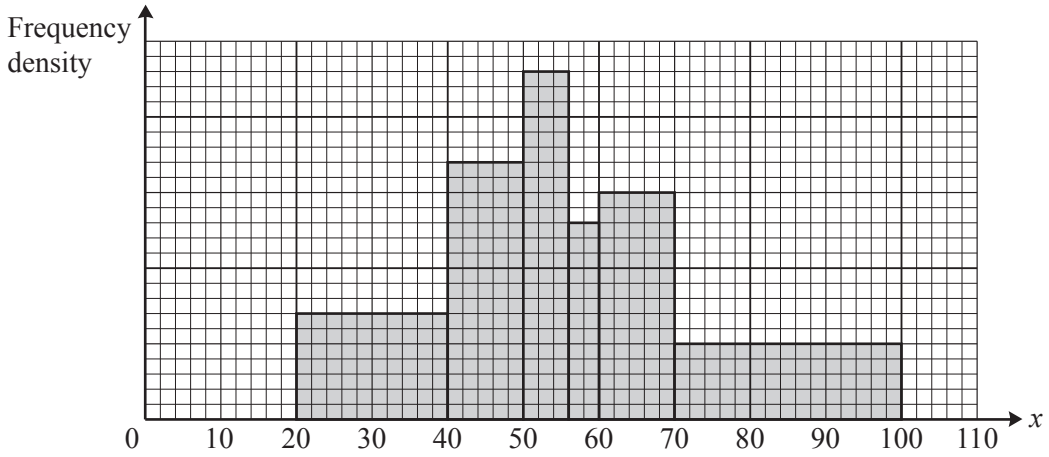
(a) $\mathrm{E}(X)$, **[1]**

(b) $\mathrm{P}(X = 2)$. **[2]**

(ii) Two values of X are chosen at random. Find the probability that their sum is less than 2. **[4]**

(iii) 10 values of X are chosen at random. Use an appropriate formula to find the probability that exactly 3 of these values are 2s. **[3]**

6 The masses, x grams, of 800 apples are summarised in the histogram.



(i) On the frequency density axis, 1 cm represents a units. Find the value of a. **[3]**

(ii) Find an estimate of the median mass of the apples. **[4]**

7 **(i)** Two judges rank n competitors, where n is an even number. Judge 2 reverses each consecutive pair of ranks given by Judge 1, as shown.

Competitor	C_1	C_2	C_3	C_4	C_5	C_6	. . . . .	C_{n-1}	C_n
Judge 1 rank	1	2	3	4	5	6	. . . . .	$n-1$	n
Judge 2 rank	2	1	4	3	6	5	. . . . .	n	$n-1$

Given that the value of Spearman's coefficient of rank correlation is $\frac{63}{65}$, find n. **[4]**

(ii) An experiment produced some data from a bivariate distribution. The product moment correlation coefficient is denoted by r, and Spearman's rank correlation coefficient is denoted by r_s.

(a) Explain whether the statement

$$r = 1 \Rightarrow r_s = 1$$

is true or false. **[1]**

(b) Use a diagram to explain whether the statement

$$r \neq 1 \Rightarrow r_s \neq 1$$

is true or false. **[2]**

8 Sandra makes repeated, independent attempts to hit a target. On each attempt, the probability that she succeeds is 0.1.

(i) Find the probability that

(a) the first time she succeeds is on her 5th attempt, **[2]**

(b) the first time she succeeds is after her 5th attempt, **[2]**

(c) the second time she succeeds is before her 4th attempt. **[4]**

Jill also makes repeated attempts to hit the target. Each attempt of either Jill or Sandra is independent. Each time that Jill attempts to hit the target, the probability that she succeeds is 0.2. Sandra and Jill take turns attempting to hit the target, with Sandra going first.

(ii) Find the probability that the first person to hit the target is Sandra, on her

(a) 2nd attempt, **[2]**

(b) 10th attempt. **[3]**

"3 sf" means "answer which rounds to … to 3 sf". Penalise over–rounding if no better answer is seen and penalise only once in the paper.

Question			Answer	Marks	Guidance	
1	(i)		$2k + 4k + 6k + 8k = 1$ $k = \frac{1}{20}$ AND $6 \times \frac{1}{20} = \frac{3}{10}$ **AG**	M1 A1 **[2]**	or $2 + 4 + 6 + 8 = 20$ M1 Must see both for A1 or $2k + 4k + 6k + 8k = 20k$ M1 $P(X = 6) = \frac{6k}{20k} = \frac{3}{10}$ A1	Must see correct wk'g for $k = \frac{1}{20}$, otherwise M0A0 NB $k \times 6 = \frac{3}{10} \Rightarrow k = \frac{1}{20}$ M0A0 (even if tested by showing that $k = \frac{1}{20}$ gives $\Sigma p = 1$) Just showing $\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = 1$ M0A0
1	(ii)		$2 \times \frac{1}{10} + 4 \times \frac{2}{10} + 6 \times \frac{3}{10} + 8 \times \frac{4}{10}$ oe $= 6$ $2^2 \times \frac{1}{10} + 4^2 \times \frac{2}{10} + 6^2 \times \frac{3}{10} + 8^2 \times \frac{4}{10}$ oe (= 10) $-$ '6'^{2} $= 4$	M1 A1 M1 M1 A1 **[5]**	$\geq$ 3 terms correct ft their values of p, dep $\Sigma p = 1$ cao $\geq$ 3 terms correct; ft their values of p; dep $\Sigma p = 1$ ft their values of p; dep +ve result & $\Sigma p = 1$ cao	Allow i.t.o. k for M1 $\div 4$ M0 Allow ito k for M1M1 $\div 4$ M0 NOT $-$ m$^2 \div 4$ $\sqrt{4} = 2$ lose final A1, not ISW, unless labelled sd
2	(i)		$\frac{3}{4} + \frac{1}{4} \times \frac{3}{8}$ $+ \frac{1}{4} \times \frac{5}{8} \times \frac{3}{16}$ $= \frac{447}{512}$ or 0.873 (3 sf)	M1 M1 A1 **[3]**	$\frac{1}{4} \times \frac{5}{8} \times \frac{13}{16}$ $(= \frac{65}{512}$ or 0.127) $1 - \frac{1}{4} \times \frac{5}{8} \times \frac{13}{16}$	
2	(ii)		0.6p or equiv seen $0.4 + 0.6p = 0.58$ $p = 0.3$	B1 M1 A1 **[3]**	Tree diag alone insufficient for mark. Or $0.6p = 0.18$. "0.18" alone insufficient	NB $0.6 \times 0.3 = 0.18$ seen at the end is probably a check, not an answer. But if 0.3 seen and 0.18 is very clearly indicated as the ans then B1M1A0

Question			Answer	Marks	Guidance	
3	(i)		$S_{xx} = 8700000 - \frac{7000^2}{6}$ (= 533333) $S_{xy} = 509900 - \frac{7000\times456}{6}$ (= –22100) $b = -\frac{"22100"}{"533333"}$ or $-\frac{663}{16000}$ (= –0.0414) $y - \frac{456}{6} =$ "–0.0414"$(x - \frac{7000}{6})$ $y = –0.0414x + 124$ (3 sf)	M1 M1 M1 A1 **[4]**	Correct subst'n in any correct *S* formula Correct subst'n in any correct *b* formula from two correct *S* formulae ft their *b* except if using *r* or $y = -\frac{663}{16000}x + \frac{3979}{32}$ or $y = –0.041x+124$	or $a = \frac{456}{6} -$ ("–0.0414")$\times \frac{7000}{6}$ oe ft "*b*" Allow y=–0.04x+124 if –0.041… seen above
3	(ii)		70 to 72	B1 **[1]**	or 71 per thousand, NOT 71000	No ft from **(i)** Ignore method
3	(iii)		Extrapolation oe Corr'n not high or small sample	B1 B1 **[2]**	Allow "2400 is beyond graph" } "Not shown on the graph" or } 1st B1 only "Line drops low, or below 0" } "Outlier" } Poor corr'n oe, or pts not close to line oe 2nd B1	"Line only allows for countries poorer than Nigeria" 1st B1 Allow "Value for Nigeria is –ve 1st B1 NOT "Other factors may apply" oe Ignore all else
3	(iv)		$S_{xx} = 8700000 + 1300^2 - \frac{(7000+1300)^2}{7}$ $S_{yy} = 36262 + 96^2 - \frac{(456+96)^2}{7}$ $S_{xy} = 509900 + 1300\times96 - \frac{8300\times552}{7}$ $r = \frac{"–19814.3"}{\sqrt{"548571"\times"1948.86"}}$ = –0.606 (3 sf)	M1 A1 M1 A1 **[4]**	or $10390000 - \frac{(8300)^2}{7} = \frac{3840000}{7}$ or 548571 or $45478 - \frac{552^2}{7} = \frac{13642}{7}$ or 1948.86 or $634700 - \frac{8300\times552}{7} = -\frac{138700}{7}$ or –19814.3 Correct subst'n in any correct *r* formula from 3 correct subs in 3 correct *S* formulae, ie all correct method	Correct sub in any correct *S* formula M1 Correct value of any *S* seen or implied by *r* A1 SC If n = 6, but otherwise correct allow M1A0M1A0 (ans r = –0.574, must see wking)

Question			Answer	Marks	Guidance	
3	(v)		No effect oe	B1 **[1]**	Stay the same oe Allow just "No"	Ignore all else
4	(i)	(a)	6	B1 **[1]**		
4	(i)	(b)	3×3×3 = 27	M1 A1 **[2]**	3! + 7×3 3 + 3×6 + 6 3! × 4 + 3 Complete correct method. Allow methods equiv to these. Only allow other methods if they appear correct	(Explanation for 3! × 4 + 3: 123: 3!, 112 & 122: 3!, 223 & 233: 3!, 331 & 311: 3! 111, 222, 333: 3 Candidates need not include this)
4	(i)	(c)	**(i)(b)** – 3 If answer is not 24, this method must be explicitly stated in order to give M1A1ft = 24 ft their **(i)(b)**	M1 A1ft **[2]**	or 3! + 6×3 or 3! + 3!×3 or 6 + 3!×3!÷2! or 3! × 4 Complete correct method. Allow methods equiv to these. Only allow other methods if they appear correct	or 8 × 3 (Explanation: there are 8 possible orders starting with 1. Candidates need not include this)
4	(ii)		eg 1123: $\frac{4!}{2!}$ × 3 alone allow M1 for $\frac{4!}{2!}$ × 3! alone eg 1122: $\frac{4!}{2!2!}$ × 3 alone allow M1 for $\frac{4!}{2!2!}$ × 3! alone Total = 54	M2 M2 A1 **[5]**	$3! \times {}^4C_1 \times 3$ or 3! × 12 M1 ÷ 2 M1dep (= 36) $3! \times {}^4C_2$ M1 ÷ 2 M1dep (= 18) Allow methods equiv to these, eg correctly listing cases Only allow other methods if they appear correct. NB 3×3×2×2 = 36 & 3×3×2 × 1 = 18 are incorrect methods unless clear justification given	This method only scores if 3×3×3×3 –…. is used: No. with 4 rep'ns = 3 M1 No. with 3 rep'ns = $\frac{4!}{3!}$ M1 × 6 (= 24) M1 or 8 × 3 M2 81–('3'+'24') or 81–27 M1 (allow 81–3 or 81–24) 18, 36 only score if a correct method seen,, or eg: 18 orders listed starting with "1" or 18 orders listed with two repetitions

Question			Answer	Marks	Guidance	
5			If incorrect p used consistently in all parts of qu 5, no mks in **(i)(a)** & **(b)** but can score M–marks in **(ii)** and **(iii)**.			
5	(i)	(a)	1.25 oe	B1 **[1]**		
5	(i)	(b)	0.8965 – 0.6328 = 0.264 (3 sf)	M1 A1 **[2]**	${}^5C_2(\frac{3}{4})^3(\frac{1}{4})^2$ $= \frac{135}{512}$ or 0.264 (3 sf)	 Answer which rounds to 0.264
5	(ii)		Answer which rounds to 0.244	M1 M1 M1 A1 **[4]**	$((\frac{3}{4})^5)^2$ or $(\frac{243}{1024})^2$ or $(\frac{3}{4})^{10}$ oe $(= \frac{59049}{1048576})$ $(\frac{3}{4})^5 \times 5(\frac{3}{4})^4(\frac{1}{4})$ or $\frac{243}{1024} \times \frac{405}{1024}$ or $5(\frac{3}{4})^9(\frac{1}{4})$ $(= \frac{98415}{1048576})$ 2×(attempt P(1, 0) alone), (NOT 2×(P(1,0) + P(0,0)) If P(sum $\leq$ 2), all three M-mks are available, but for 3rd M1, must be 2×(P(1,0)+P(2,0)) only Ans 0.150 probably M1M1M0A0 but check working Ans 0.188 probably M0M1M1A0 but check working	B(10. 0.25) seen or implied M1 Table or formula with n = 10 used M1 P($X \leq 1$) from table or $(\frac{3}{4})^{10} + 10(\frac{3}{4})^9 \times (\frac{1}{4})$ M1 0.244 (3 sf) A1 P($X \leq 2$) = 0.526 from table n = 10 M1M1M1A0 SC P(X = 2) answer 0.282: B1
5	(iii)		Use of 0.2637 or 0.264 ${}^{10}C_3 \times (1 - \text{'0.2637'})^7 \times \text{'0.2637'}^3$ = 0.258 (3 sf)	M1 M1 A1 **[3]**	or their **(i)(b)** ft **(i)(b)** allow ft their **(ii)** for this M1 only Correct ans, no working: M1M1A1	SC allow ${}^{10}C_3 \times (1 - \text{'0.282'})^7 \times \text{'0.282'}^3$ M0M1 (0.282 comes from P(3 totals = 2))

Question			Answer	Marks	Guidance	
6	(i)		Attempt find total area, (even if includes a^2) eg $20\times1.4a+10\times3.4a+6\times4.6a+4\times2.6a+10\times3a+30a$ or $28a+34a+27.6a+10.4a+30a+30a$ or $20\times1.4+10\times3.4+6\times4.6+4\times2.6+10\times3+30$ or $28+34+27.6+10.4+30+30$ or $7\times20+17\times10+23\times6$ + …… or $160a$ or 160 or 16 or $16a$ (if area, not ht) 800 ÷ their total (must involve area, not ht) eg $160a = 800$, 800÷ $a = 5$ "Box" ⇒ area. "Square" possibly ⇒ area	M1 M1dep A1 **[3]**	eg tot area = $16cm^2$ or $16a$ M1 800/16 (= 50) M1 $a \times 10 = 50$ $a = 5$ A1 eg tot area = 400 (sqs) M1 800/400 (= 2) M1 $1.4a \times 20 = 70 \times 2$ $a = 5$ A1 Correct ans with nothing incorrect seen: M1M1A1 But where the correct answer clearly results from incorrect working, eg $a = 800/167 = 4.8$ rounded to $a = 5$, then max M1M1A0	Trial methods, eg: $a = 5$ gives $7\times20+17\times10+23\times6$ + …. = 800 M1 But no of apples = 800 M1 Hence $a = 5$ A1 $a = 10$ gives $14\times20+34\times10+46\times6$+. = 1600 M1 But no of apples = 800 M1 Hence $a = 5$ A1 NOT "1cm = 5" (because may just come from counting squares) NB total ht = 16cm so if 16 seen, must clearly be area eg 800/16 may score 0 or 2
6	(ii)		$\frac{1}{2}$ total area or $\frac{1}{2}$ total no. apples ft their 6(i) Median is in 50 – 56 class stated or implied Calculate (approx) $\frac{2}{3}$ of way along class or $\frac{1}{3}$ of way from top of class Median = 53.9 to 54 Not eg 54.2	B1f M1 M1 A1 **[4]**	Correct ans with nothing incorrect seen: M1M1A1 But where the correct answer clearly results from incorrect working, eg $a = 800/167 = 4.8$ rounded to $a = 5$, then max M1M1A0	Examples of correct methods: $400 - (7\times20+17\times10)$ (= 90) $50 + \frac{"90"}{23\times6} \times 6 = 54$ $200 - (70+85)$ (= 45) $50 + \frac{"45"}{69} \times 6 = 54$ $400.5 - (7\times20+17\times10)$ (= 90.5) $50 + \frac{"90.5"}{23\times6} \times 6 = 54$ Use of LB = 49.5: eg median = 49.5 +appr $\frac{2}{3} \times 6 = 53.4$ B1M1A1A0

Question			Answer	Marks	Guidance	
7	(i)		$\Sigma d^2 = n$ seen or implied $1 - \frac{6\times\text{anything}}{n(n^2-1)} = \frac{63}{65}$ or $\frac{6\times\text{anything}}{n(n^2-1)} = \frac{2}{65}$ $\frac{6}{(n^2-1)} = \frac{2}{65}$ or eg $390 = 2(n^2 - 1)$ $n = 14$ NOT $n = \pm 14$	M1 M1 A1 depM2 A1 **[4]**	eg $1-\frac{6\times\Sigma d^2}{n(n^2-1)}$ or $1-\frac{6\times n^2}{n(n^2-1)}$ or $1-\frac{6\times 1^n}{n(n^2-1)}$ or $1-\frac{6\times 6^2}{n(n^2-1)} = \frac{63}{65}$ Any correct eqn after cancelling n or take out factor of n; can be implied by $n = 14$ But A0 if $n = 14$ clearly follows from incorrect working If no working or unclear working, but n = 14, M1M1A1A1	Trial method: $\Sigma d^2 = 14$ M1 $1 - \frac{6\times 14}{14(14^2-1)}$ oe M1 $= \frac{63}{65}$ A1 (0.969 : A0) $\Rightarrow n = 14$ A1 Conclusion needed
7	(ii)	(a)	$r = 1 \Rightarrow$ st line, hence true (or $r_s = 1$) oe Explanation essential Must state or imply "true"	B1 **[1]**	$r = 1 \Rightarrow y$ incr as x incr, so $r_s = 1$ oe Allow "True because perfect corr'n" or "True because $r = 1$ means pts ranked in order so $r_s = 1$" "$r = 1$ means the ranks will agree" "$r = 1$ means all d's are 0, hence $r_s = 1–0 = 1$"	NOT "r incr so ranks incr" NOT "$r_s = r$ for ranks so true" NOT "True because strong corr'n"
7	(ii)	(b)	Diag, $\geq$ 3 pts, not on st line but with $x_{n+1}>x_n$ & $y_{n+1}>y_n$, Zig zag line or curve, moving up & right so r_s can still be 1 eg "expon'l curve gives $r \neq 1$ but $r_s = 1$" B1B1	B1 B1dep **[2]**	Ignore explan if correct diag given Ignore any st line drawn Allow numerical example for which $r \neq 1$ but $r_s = 1$. If expl'n contradicts diag, mark diag For 2nd B1 must state or imply "false"	

Question			Answer	Marks	Guidance	
8	(i)	(a)	$0.9^4 \times 0.1$ $= \frac{6561}{100000}$ or 0.0656 (3sf)	M1 A1 **[2]**		
8	(i)	(b)	0.9^5 $= \frac{59049}{100000}$ or 0.59 (2 sf)	M1 A1 **[2]**	Allow 0.9^4 or $1–0.9^5$:M1 but $1–0.9^n$ ($n \neq 5$) or 0.1×0.9^n :M0	$1 – (0.1+0.9\times0.1+0.9^2\times0.1 + \ldots 0.9^4\times0.1)$ Allow without "1 –" OR omit last term NB $0.9^5\times0.1 = 0.0590$ M0A0
8	(i)	(c)	0.1×0.1 or $[0.1\times0.1\times0.9 + 0.1\times0.1\times0.1]$ oe $+ 0.1\times0.9\times0.1$ oe $+ 0.9\times0.1\times0.1$ oe $= 0.028$	M1 M1 M1 A1 **[4]**	M1M1 two correct terms, no incorrect multiples M1 all correct Ans 0.027 probably M0M1M1A0 but check working SC if no M-mks scored: SSF, SSS, FSS, SFS or SS, FSS, SFS seen or implied: B1	$3\times0.1^2\times0.9+0.1^3$ no incorrect multiples M2 for 1st term; M1 for 2nd This method only scores using "1 – ": 0.9^3; $3\times0.9^2\times0.1$ no incorrect multiples M1; M1 1 – one or both terms with no further wking: M1(dep M1) eg $1 – 0.9^3$ alone M1M0M1
8	(ii)	(a)	$0.9\times 0.8 \times 0.1$ $= \frac{9}{125}$ or 0.072	M1 A1 **[2]**	alone or allow × 0.8 (ie girls in wrong order) (= 0.0576)	NOT $0.9\times0.8\times0.1\times0.2= 0.0144$: M0A0 NOT $0.9\times0.8\times0.2= 0.144$: M0A0
8	(ii)	(b)	$0.9^{9 \text{ or } 10} \times 0.8^{9 \text{ or } 10} \times 0.1$ (or ×0.2, not ×0.1×0.2) $(0.9 \times 0.8)^9 \times 0.1$ oe $= 5.2\times10^{-3}$ or 0.0052 (2 sf)	M1 M1 A1 **[3]**	allow $0.9^{9 \text{ or } 10} \times 0.8^{9 \text{ or } 10} \times 0.1 \times {}^{18,19,20}C_1$ fully correct SC Consistent use of 0.8 for both girls: **(ii)(a)** 0.128 **(ii)(b)** 0.00360 or 0.9 for both girls: **(ii)(a)** 0.081 **(ii)(b)** 0.0150 If both these ans seen, allow (a) 0 (b) B1	If ans = 0.00360 or 0.0150 see SC below